Units and Measurements

Quick Revision

1. **Physical Quantities** All the quantities which can be measured directly or indirectly and in terms of which laws of Physics are described are called physical quantities.

   These can be divided into two types, namely **fundamental** and **derived quantities.**

   - **Fundamental Quantities** The physical quantities which are independent of other physical quantities and are not defined in terms of other physical quantities are called fundamental or base quantities.
     e.g. Mass, length, time, temperature, luminous intensity, electric current, amount of substance, etc.
   - **Derived Quantities** Those quantities which can be derived from the fundamental physical quantities are called derived quantities.
     e.g. Velocity, acceleration, linear momentum, etc.

2. **Physical Unit** The standard amount of a physical quantity chosen to measure the physical quantity of same kind is called a physical unit. The physical units can be classified into following two types

   - **Fundamental Units** The units of fundamental quantities are known as fundamental units.
   - **Derived Units** The units of measurement of all other physical quantities, which can be obtained from fundamental units are called derived units.

3. **System of Units** It is the complete set of units, both fundamental and derived physical units.

   The common system of units used in mechanics are as follows

   - **The FPS System** It is the British engineering system of units. It uses foot as the unit of length, pound as the unit of mass and second as the unit of time.
   - **The CGS System** It is the French system of units, which uses centimetre, gram and second as the units of length, mass and time, respectively.
   - **The MKS System** It uses metre, kilogram and second as the fundamental units of length, mass and time, respectively.
   - **The International System of Units** (SI Units) The system of units which is accepted internationally for measurement is the 'Systeme International d' Units (French for International System of Units), abbreviated as SI.

4. **Fundamental Quantities and their SI Units**

Fundamental quantity	SI unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

5. **Supplementary Quantities and their SI Units**

Supplementary quantity	SI unit	Symbol
Plane angle	radian	rad
Solid angle	steradian	sr

6. **Some Important Practical Units**

For Length/Distance

- **Astronomical Unit** It is the mean distance of the earth from the sun.

$$1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$$

- **Light year** It is the distance travelled by light in vacuum in one year.

$$1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$$

- **Micro or micrometer**, $1 \mu\text{m} = 10^{-6}$ m
- **Nanometer**, $1 \text{ nm} = 10^{-9}$ m
- **Angstrom**, $1 \text{ Å} = 10^{-10}$ m
- **Fermi** This unit is used for measuring nuclear sizes. $1 \text{ Fm} = 10^{-15}$ m

For Mass

- **Pound**, 1 lb = 0.4536 kg
- **Slug**, 1 slug = 14.59 kg
- **Quintal**, 1 q = 100 kg
- **Tonne** or **metric tonne**, 1 t = 1000 kg
- **Atomic mass unit** (It is defined as the (1/12)th of the mass of one $^{12}_{6}$C-atom) 1 u or amu $= 1.66 \times 10^{-27}$ kg.

For Area

- **Barn**, 1 barn $= 10^{-28}$ m^2
- **Acre**, 1 acre = 4047 m^2
- **Hectare**, 1 hectare $= 10^4$ m^2

7. **Accuracy and Precision of Instruments**
The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity. While, precision tells us to what resolution or limit the quantity is measured.

8. **Errors in Measurement**

Difference in the true value and the measured value of a quantity is called error in measurement.

Error = True value − Measured value

In general, the errors can be further classified as

- **Systematic Errors** Those errors that tend to be in one direction, either positive or negative are called systematic errors. Some of the sources of systematic errors are
  - (a) **Instrumental errors** They occur due to imperfect design or manufacture or calibration of the measuring instrument.
  - (b) **Imperfection in experimental technique or procedure** These types of errors occur due to the experimental arrangement limitations.
  - (c) **Personal errors** These errors arise due to inexperience of the observer such as lack of proper setting of the apparatus and taking observations without observing proper precautions, etc.
  - (d) **Errors due to external causes** Various parameters such as change in temperature, pressure, volume, etc. during experiment may affect the reading of measurement.
- **Random Errors** The errors which occur irregularly and are random in magnitude and direction are called random errors.
- **Least Count Error** The smallest value that can be measured by a measuring instrument is called the least count of the instrument and error in its value is called least count error.

- **Absolute Error** The magnitude of the difference between the true value of the quantity and the individual measured value is called the absolute error of the measurement. It is denoted by $|\Delta a|$.

  Suppose, the measured values are $a_1, a_2, a_3, \ldots, a_n$, then arithmetic mean of these values is $a_{\text{Mean}} = \dfrac{a_1 + a_2 + \ldots + a_n}{n}$.

  If we take arithmetic mean a_{Mean} as the true value, then the absolute errors in the individual measured value will be

  $$\Delta a_1 = a_{\text{Mean}} - a_1$$

- **Mean Absolute Error** It is the arithmetic mean of the magnitudes of absolute errors in all the measurements of the quantity.

  Mean absolute error, $\Delta a_{\text{Mean}} = \sum_{i=1}^{n} \dfrac{|\Delta a_i|}{n}$

- **Relative Error or Fractional Error** It is defined as the ratio of mean absolute error to the mean value of the quantity measured.

  Relative error, $\delta a = \dfrac{\Delta a_{\text{Mean}}}{a_{\text{Mean}}}$

- **Percentage Error** When fractional error or relative error is expressed in percentage, then it is called percentage error.

  Percentage error, $\delta a\,\% = \dfrac{\Delta a_{\text{Mean}}}{a_{\text{Mean}}} \times 100\%$

9. **Combination of Errors**

- **Error in Sum or Difference**

  Let $X = A + B$ or $X = A - B$

  where, A and B are physical quantities have measured value $A \pm \Delta A$, $B \pm \Delta B$, respectively.

  So, the maximum possible error in sum and difference, $\Delta Z = \Delta A + \Delta B$

- **Error in Product or Quotient**

  Let $X = A \times B$ or $X = \dfrac{A}{B}$

  So, the maximum possible error in product or quotient is $\dfrac{\Delta Z}{Z} = \dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}$

- **Error in Case of a Measured Quantity Raised to a Power**

  Relative error of $Z = A^n B^m$ is

  $$\frac{\Delta Z}{Z} = \pm \left[n \frac{\Delta A}{A} + m \frac{\Delta B}{B} \right]$$

10. **Significant Figures**

The digits that are known reliably plus the first uncertain digit are known as significant digits or significant figures.

Rules for Determining the Number of Significant Figures

***Rule* 1** All non-zero digits are significant.

***Rule* 2** All the zeros between two non-zero digits are significant, no matter where the decimal point is, if at all.

***Rule* 3** If the number is less than one, the zero(s) on the right of decimal point and to the left of first non-zero digit are not significant.

***Rule* 4** In a number without a decimal point, the terminal or trailing zeros is not significant.

***Rule* 5** The trailing zero(s) in a number with a decimal points are significant.

11. **Rules for Arithmetical Operations with Significant Figures**

Some rules of arithmetical operations with significant figures are as given below

- **Addition and Subtraction** In both, addition and subtraction, the final result should retain as **many decimal places** as are there in the number with the **least decimal places**.
- **Multiplication and Division** In multiplication or division, the final result should retain as many significant figures, as are there in the original number with the least significant figures.

12. **Rounding Off** The result of computation with approximate numbers, which contains more than one uncertain digit, should be rounded off. While rounding off measurements, we use the following rules by convention

***Rule* 1** If the digit to be dropped is less than 5, then the preceding digit is left unchanged.

***Rule* 2** If the digit to be dropped is more than 5, then the preceding digit is raised by one.

***Rule* 3** If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by one.

***Rule* 4** If the digit to be dropped is 5 and followed by zeros, then the preceding digit is left unchanged, if it is even.

***Rule* 5** If the digit to be dropped is 5 and followed by zeros, then the preceding digit is raised by one, if it is odd.

13. **Dimensions of a Physical Quantity**
The dimensions of a physical quantity are the powers (or exponents) to which the units of base quantities are raised to represent a derived unit of that quantity. There are seven base quantities represented with square brackets [ ] such as length [L], mass [M], time [T], electric current [A], thermodynamic temperature [K], luminous intensity [cd] and amount of substances [mol].

14. **Dimensional Formulae and Dimensional Equations**
The expression which shows how and which of the fundamental quantities represent the dimension of the physical quantity is called the **dimensional formula** of the given physical quantity.

Some of the dimensional formulae are as given below

Acceleration = $[M^0\ L^1\ T^{-2}]$

Mass density = $[ML^{-3}\ T^0]$

Volume = $[M^0 L^3 T^0]$

The equation obtained by equating a physical quantity with its dimensional formula is called the **dimensional equation** of the given physical quantity.

15. **Dimensional Analysis and its Applications**
The dimensional analysis helps us in deducing the relations among different physical quantities and checking the accuracy, derivation and dimensional consistency or homogeneity of various numerical expressions. Its applications are as given below

- **Checking the Dimensional Consistency of Equations** The principle of homogeneity of dimension states that, a physical quantity equation will be dimensionally correct, if the dimensions of all the terms occurring on both sides of the equation are same.
- **Conversion of One System of Units into Another** If M_1, L_1 and T_1 are the fundamental units of mass, length and time in one system and while for other system, M_2, L_2 and T_2 are the fundamental units of mass, length and time, then $n_1 = [M_1^a\ L_1^b\ T_1^c]$ and $n_2 = [M_2^a\ L_2^b\ T_2^c]$.

  From $n_1 u_1 = n_2 u_2$, where u_1 and u_2 are two units of measurement of the quantity and n_1 and n_2 are their respective numerical values.

  $$\text{Then, } n_2 = \frac{n_1\ [M_1^a L_1^b T_1^c]}{[M_2^a L_2^b T_2^c]}$$

  $$= n_1 \left[\frac{M_1}{M_2}\right]^a \cdot \left[\frac{L_1}{L_2}\right]^b \cdot \left[\frac{T_1}{T_2}\right]^c$$

- **Deducing Relation among the Physical Quantities** The method of dimensions is used to deduce the relation among the physical quantities. We should know the dependence of the physical quantity on other quantities.

Objective Questions

Multiple Choice Questions

1. The quantity having the same unit in all system of unit is

(a) mass (b) time
(c) length (d) temperature

2. The SI unit of thermal conductivity is

(a) $J\ m^{-1}K^{-1}$ (b) $W\text{-}m\ K^{-1}$
(c) $W\ m^{-1}K^{-1}$ (d) $Jm\ K^{-1}$

3. The damping force on an oscillator is directly proportional to the velocity. The unit of the constant of proportionality is

(a) $kg\text{-}ms^{-1}$ (b) $kg\text{-}ms^{-2}$
(c) kgs^{-1} (d) kg-s

4. The density of a material in SI units is $128\ kg\ m^{-3}$. In certain units in which the unit of length is 25 cm and the unit of mass is 50 g, the numerical value of density of the material is

(a) 40 (b) 16 (c) 640 (d) 410

5. If the value of work done is $10^{10}\ g\text{-}cm^2\ s^{-2}$, then its value in SI units will be

(a) $10\ kg\text{-}m^2 s^{-2}$ (b) $10^2\ kg\text{-}m^2 s^{-2}$
(c) $10^4\ kg\text{-}m^2 s^{-2}$ (d) $10^3\ kg\text{-}m^2 s^{-2}$

6. Amongst the following options, which is a unit of time?

(a) Light year (b) Parsec
(c) Year (d) None of these

7. The moon is observed from two diametrically opposite points *A* and *B* on earth. The angle θ subtended at the moon by the two directions of observation is 1°54′; given that the diameter of the earth to be about 1.276×10^7 m. Compute the distance of the moon from the earth.

(a) 4.5×10^9 m
(b) 3.83×10^8 m
(c) 2.5×10^4 m
(d) 4×10^7 m

8. The ratio of the volume of the atom to the volume of the nucleus is of the order of

(a) 10^{15} (b) 10^{25}
(c) 10^{20} (d) 10^{10}

9. Which of the following measurement is most precise? *(NCERT Exemplar)*

(a) 5.00 mm (b) 5.00 cm
(c) 5.00 m (d) 5.00 km

10. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it?

(a) A meter scale
(b) A vernier calliper where the 10 divisions in vernier scale matches with 9 divisions in main scale and main scale has 10 divisions in 1 cm
(c) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm
(d) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm

11. The length, breadth and height of a rectangular block of wood were measured to be $l = 12.13 \pm 0.02$ cm, $b = 8.16 \pm 0.01$ cm and $h = 3.46 \pm 0.01$ cm.

(a) 0.88% (b) 0.58%
(c) 0.78% (d) 0.68%

12. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90s, 91s, 92s and 95s. If the minimum division in the measuring clock is 1s, then the reported mean time should be

(a) (92 ± 2)s (b) (92 ± 5)s
(c) (92 ± 1.8)s (d) (92 ± 3)s

13. In successive experiments while measuring the period of oscillation of a simple pendulum. The readings turn out to be 2.63 s, 2.56 s, 2.42 s, 2.71s and 2.80 s. Calculate the mean absolute error.

(a) 0.11 s (b) 0.42 s (c) 0.92 s (d) 0.10 s

14. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{L/g}$. Measured value of L is 20 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1s resolution.What is the percentage error in the determination of g?

(a) 5% (b) 3%
(c) 4% (d) 7%

15. Calculate the mean percentage error in five observations,

80.0, 80.5, 81.0, 81.5 and 82.

(a) 0.74% (b) 1.74%
(c) 0.38% (d) 1.38%

16. Calculate the relative errors in measurement of two masses 1.02 g $\pm$ 0.01g and 9.89g $\pm$ 0.01g .

(a) $\pm$ 1% and $\pm$ 0.2% (b) $\pm$ 1% and $\pm$ 0.1%
(c) $\pm$ 2% and $\pm$ 0.3% (d) $\pm$ 3% and $\pm$ 0.4%

17. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is

(a) 2.5% (b) 3.5% (c) 4.5% (d) 6%

18. Percentage errors in the measurement of mass and speed are 2% and 3%, respectively. The error in the estimation of kinetic energy obtained by measuring mass and speed will be

(a) 8% (b) 2%
(c) 12% (d) 10%

19. If the length of a pendulum is increased by 2%, then in a day, the pendulum

(a) loses 764 s (b) loses 924 s
(c) gains 236 s (d) loses 864 s

20. The length and breadth of a rectangular sheet are 16.2 cm and 10.1 cm, respectively. The area of the sheet in appropriate significant figures and error is *(NCERT Exemplar)*

(a) 164 $\pm$ 3 cm^2 (b) 163.62 $\pm$ 2.6 cm^2
(c) 163.6 $\pm$ 2.6 cm^2 (d) 163.62 $\pm$ 3 cm^2

21. In an experiment, four quantities a, b, c and d are measured with percentage error 1%, 2%, 3% and 4%, respectively. Quantity P is calculated as follows $P = \frac{a^3 b^2}{cd}$, percentage error in P is

(a) 14% (b) 10% (c) 7% (d) 4%

22. A physical quantity z depends on four observables a, b, c and d, as $z = \frac{a^2 b^{2/3}}{\sqrt{c} d^3}$.

The percentages of error in the measurement of a, b, c and d are 2%, 1.5%, 4% and 2.5% respectively. The percentage of error in z is

(a) 13.5% (b) 16.5% (c) 14.5% (d) 12.25%

23. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are

(a) 5, 1, 2 (b) 5, 1, 5
(c) 5, 5, 2 (d) 4, 4, 2

24. If 3.8×10^{-6} is added to 42×10^{-6} giving due regard to significant figures, then the result will be

(a) 4.58×10^{-5} (b) 4.6×10^{-5}
(c) 45×10^{-5} (d) None of these

25. The numbers 5.355 and 5.345 on rounding off to 3 significant figures will give

(a) 5.35 and 5.34 (b) 5.36 and 5.35
(c) 5.35 and 5.35 (d) 5.36 and 5.34

26. The mass and volume of a body are 4.237 g and 2.5 cm^3, respectively. The density of the material of the body in correct significant figures is *(NCERT Exemplar)*

(a) 1.6048 $g\,cm^{-3}$ (b) 1.69 $g\,cm^{-3}$
(c) 1.7 $g\,cm^{-3}$ (d) 1.695 $g\,cm^{-3}$

27. If mass M, distance L and time T are fundamental quantities, then find the dimensions of torque.

(a) $[ML^2T^{-2}]$ (b) $[MLT^{-2}]$
(c) $[MLT]$ (d) $[ML^2T]$

28. Let l, r, c and v represent inductance, resistance, capacitance and voltage, respectively. The dimension of $\frac{l}{rcv}$ in SI units will be

(a) $[LT^2]$ (b) $[LTA]$
(c) $[A^{-1}]$ (d) $[LA^{-2}]$

29. Obtain the dimensional formula for universal gas constant.

(a) $[ML^2T^{-2}\,mol^{-1}\,K^{-1}]$
(b) $[ML^3T^{-1}\,mol^{-2}\,K^{-2}]$
(c) $[M^2LT^{-1}\,mol^{-1}\,K^{-1}]$
(d) $[M^3LT^{-2}\,mol^{-1}\,K^{-2}]$

30. Which two of the following five physical parameters have the same dimensions?

I. Energy density
II. Refractive index
III. Dielectric constant
IV. Young's modulus
V. Magnetic field

(a) I and IV (b) III and V
(c) I and II (d) I and V

31. If P, Q, R are physical quantities, having different dimensions, which of the following combinations can never be a meaningful quantity?

(a) $(P-Q)/R$ (b) $PQ-R$
(c) PQ/R (d) $(PR-Q^2)/R$

32. The potential energy of a particle varies with distance x from a fixed origin as $U = \frac{A\sqrt{x}}{x+B}$, where A and B are constants. The dimensions of AB are

(a) $[ML^{5/2}T^{-2}]$ (b) $[ML^2T^{-2}]$
(c) $[M^{3/2}L^3T^{-2}]$ (d) $[ML^{7/2}T^{-2}]$

33. In the formula, $X = 3YZ^2$, X and Z have dimensions of capacitance and magnetic induction. The dimensions of Y in MKSQ system are

(a) $[M^{-3}L^{-2}T^4Q^4]$ (b) $[ML^2T^8Q^4]$
(c) $[M^{-2}L^{-3}T^2Q^4]$ (d) $[M^{-2}L^{-2}TQ^2]$

34. If the velocity v (in cms^{-1}) of a particle is given in terms of t (in second) by the relation $v = at + \frac{b}{t+c}$, then the dimensions of a, b and c are

	a	b	c
(a)	$[L]$	$[LT]$	$[T^2]$
(b)	$[L^2]$	$[T]$	$[LT^{-2}]$
(c)	$[LT^2]$	$[LT]$	$[L]$
(d)	$[LT^{-2}]$	$[L]$	$[T]$

35. A book with many printing errors contains four different formulae for the displacement y of a particle under going a certain periodic motion,

where, a = maximum displacement of the particle, v = speed of the particle, T = time period of motion.
Which are the correct formulae on dimensional grounds?

(a) $y = a\sin\frac{2\pi t}{T}$ (b) $y = a\sin vt$
(c) $y = \left(\frac{a}{T}\right)\sin(t/a)$ (d) None of these

36. If speed V, area A and force F are chosen as fundamental units, then the dimensional formula of Young's modulus will be

(a) $[FA^2V^{-3}]$ (b) $[FA^{-1}V^0]$
(c) $[FA^2V^{-2}]$ (d) $[FA^2V^{-1}]$

37. If dimensions of critical velocity v_c of a liquid flowing through a tube are expressed as $[\eta^x \rho^y r^z]$, where η, ρ and r are the coefficient of viscosity of liquid, density of liquid and radius of the tube respectively, then the values of x, y and z are given by

(a) 1, –1, –1 (b) –1, –1, 1
(c) –1, –1, –1 (d) 1, 1, 1

38. The density of a material in CGS system is 10 g cm^{-3}. If unit of length becomes 10 cm and unit of mass becomes 100 g, the new value of density will be

(a) 10 units (b) 100 units
(c) 1000 units (d) 1 unit

39. When 1 m, 1 kg and 1 min are taken as the fundamental units, the magnitude of the force is 36 units. What will be the value of this force in CGS system?

(a) 10^5 dyne (b) 10^3 dyne
(c) 10^8 dyne (d) 10^4 dyne

40. The solid angle subtended by the periphery of an area $1\ cm^2$ at a point situated symmetrically at a distance of 5 cm from the area is ……… steradian.

(a) 2×10^{-2} (b) 4×10^{-2} (c) 6×10^{-2} (d) 8×10^{-2}

41. Measure of two quantities along with the precision of respective measuring instrument is $A = 2.5\ ms^{-1} \pm 0.5\ ms^{-1}$, $B = 0.10\ s \pm 0.01\ s$. The value of AB will be ……… . *(NCERT Exemplar)*

(a) (0.25 ± 0.08) m (b) (0.25 ± 0.5) m
(c) (0.25 ± 0.05) m (d) (0.25 ± 0.135) m

42. It is claimed that two cesium clocks, if allowed to run for 100 yrs without any disturbance may differ by only about 0.02 s. Then the accuracy of the clock in measuring a time interval of 1 s is ……… .

(a) 10^{-10} (b) 10^{-11}
(c) 10^{-5} (d) 10^{-8}

43. Photon is quantum of radiation with energy $E = h\nu$, where ν is frequency and h is Planck's constant. The dimensions of h are the same as that of ……… .

(a) linear impulse
(b) angular impulse
(c) linear momentum
(d) energy

44. Which amongst the following statement is incorrect regarding mass?

(a) Its SI unit is kilogram.
(b) It does not depend on the location of the object in space.
(c) It is the basic property of matter.
(d) While dealing with atoms, kilogram is a convenient unit for measuring mass.

45. Choose the incorrect statement out of the following.

(a) Every measurement by any measuring instrument has some errors.
(b) Every calculated physical quantity that is based on measured values has some errors.
(c) A measurement can have more accuracy but less precision and *vice-versa*.
(d) The percentage error is different from relative error.

46. Given that T stands for time period and l stands for the length of simple pendulum. If g is the acceleration due to gravity, then which of the following statements about the relation $T^2 = l/g$ is correct?

(a) It is correct both dimensionally as well as numerically.
(b) It is neither dimensionally correct nor numerically.
(c) It is dimensionally correct but not numerically.
(d) It is numerically correct but not dimensionally.

47. Match the following columns.

	Column I		Column II
A.	Capacitance	p.	volt $(ampere)^{-1}$
B.	Magnetic induction	q.	volt-sec $(ampere)^{-1}$
C.	Inductance	r.	newton $(ampere)^{-1}$ $(metre)^{-1}$
D.	Resistance	s.	$coulomb^2 (joule)^{-1}$

Codes

	A	B	C	D
(a)	q	r	s	p
(b)	s	r	q	p
(c)	r	s	p	q
(d)	s	p	q	r

48. Match the Column I (unit) with Column II (value) and select the correct option from the codes given below.

	Column I		Column II
A.	1 are	p.	200 mg
B.	1 bar	q.	1.013×10^5 Pa
C.	1 carat	r.	10^2 m^2

Codes

	A	B	C		A	B	C
(a)	q	p	r	(b)	r	r	p
(c)	r	q	p	(d)	r	p	q

49. Names of units of some physical quantities are given in Column I and their dimensional formulae are given in Column II and select the correct option from the codes given below.

	Column I		Column II
A.	Pa-s	p.	$[L^2T^{-2}K^{-1}]$
B.	Nm-K^{-1}	q.	$[MLT^{-2}A^{-1}K^{-1}]$
C.	J kg^{-1} K^{-1}	r.	$[ML^{-1}T^{-1}]$
D.	Wb m^{-1} K^{-1}	s.	$[ML^2T^{-2}K^{-1}]$

Codes

	A	B	C	D
(a)	s	r	p	q
(b)	r	q	s	p
(c)	r	p	s	q
(d)	r	s	p	q

Assertion-Reasoning MCQs

*For question numbers 50 to 59, two statements are given-one labelled **Assertion** (A) and the other labelled **Reason** (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) are as given below*

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

50. Assertion Unit chosen for measuring physical quantities should not be easily reproducible.

Reason Unit should change with the changing physical conditions like temperature, pressure, etc.

51. Assertion The unit used for measuring nuclear cross-section is 'barn'.

Reason 1 barn = 10^{-14} m^2.

52. Assertion When we change the unit of measurement of a quantity, its numerical value changes.

Reason Smaller the unit of measurement smaller is its numerical value.

53. Assertion Parallax method is used for measuring distances of nearby stars only.

Reason With increase in the distance of star from earth, the parallactic angle becomes too small to be measured accurately.

54. **Assertion** Out of two measurements $l = 0.7$ m and $l = 0.70$ m, the second one is more accurate.

Reason In every measurement, the last digit is not accurately know.

55. **Assertion** Random errors arise due to random and unpredictable fluctuations in experimental conditions.

Reason Random errors occurred due to irregularly with respect to sign and size.

56. **Assertion** When a quantity appears with a power n greater than one in an expression, its error contribution to the final result decreases n times.

Reason In all mathematical operations, the errors are not additive in nature.

57. **Assertion** Special functions such as trigonometric, logarithmic and exponential functions are not dimensionless.

Reason A pure number, ratio of similar physical quantities, such as angle and refractive index, has some dimensions.

58. **Assertion** Specific gravity of a fluid is a dimensionless quantity.

Reason It is the ratio of density of fluid to the density of water.

59. **Assertion** The method of dimensions analysis cannot validate the exact relationship between physical quantities in any equation.

Reason It does not distinguish between the physical quantities having same dimensions.

Case Based MCQs

Direction Answer the questions from 60-64 on the following case.

Measurement of Physical Quantity

All engineering phenomena deal with definite and measured quantities and so depend on the making of the measurement. We must be clear and precise in making these measurements. To make a measurement, magnitude of the physical quantity (unknown) is compared.

The record of a measurement consists of three parts, i.e. the dimension of the quantity, the unit which represents a standard quantity and a number which is the ratio of the measured quantity to the standard quantity.

60. A device which is used for measurement of length to an accuracy of about 10^{-5} m, is

(a) screw gauge (b) spherometer
(c) vernier callipers (d) Either (a) or (b)

61. Which of the technique is not used for measuring time intervals?

(a) Electrical oscillator
(b) Atomic clock
(c) Spring oscillator
(d) Decay of elementary particles

62. The mean length of an object is 5 cm. Which of the following measurements is most accurate?

(a) 4.9 cm (b) 4.805 cm
(c) 5.25 cm (d) 5.4 cm

63. If the length of rectangle $l = 10.5$ cm, breadth $b = 2.1$ cm and minimum possible measurement by scale = 0.1 cm, then the area is

(a) 22.0 cm^2 (b) 21.0 cm^2
(c) 22.5 cm^2 (d) 21.5 cm^2

64. Age of the universe is about 10^{10} yr, whereas the mankind has existed for 10^{6} yr. For how many seconds would the man have existed, if age of universe were 1 day?

(a) 9.2 s (b) 10.2 s
(c) 8.6 s (d) 10.5 s

Direction *Answer the questions from 65-69 on the following case.*

Significant Digits

Normally, the reported result of measurement is a number that includes all digits in the number that are known reliably plus first digit that is uncertain. The digits that are known reliably plus the first uncertain digit are known as significant digits or significant figures.

e.g. When a measured distance is reported to be 374.5 m, it has four significant figures 3, 7, 4 and 5. The figures 3, 7, 4 are certain and reliable, while the digit 5 is uncertain. Clearly, the digits beyond the significant digits reported in any result are superfluous.

65. In 4700 m, significant digits are

(a) 2 (b) 3 (c) 4 (d) 5

66. To determine the number of significant figures, scientific notation is

(a) a^b (b) $a \times 10^b$
(c) $a \times 10^2$ (d) $a \times 10^4$

67. 5.74 g of a substance occupies $1.2\ \text{cm}^3$. Express its density by keeping the significant figures in view.

(a) $4.9\ \text{g cm}^{-3}$ (b) $5.2\ \text{g cm}^{-3}$
(c) $4.8\ \text{g cm}^{-3}$ (d) $4.4\ \text{g cm}^{-3}$

68. Choose the correct option.

(a) Change in unit does not change the significant figure.
(b) 4.700 m = 4700 mm, here there is a change of significant number from 4 to 2 due to change in unit.
(c) $4700\ \text{m} = 4.700 \times 10^3\ \text{m}$, here there is change in numbers of significant numbers.
(d) Change in unit changes the number of significant figure.

69. Consider the following rules of significant figures.

I. All the non-zero digits are significant.
II. All the zeroes between two non-zero digits are significant.
III. The terminal or trailing zero(s) in a number without a decimal point are significant.

Which of the above statement(s) is/are correct?

(a) I and II (b) II and III
(c) I and III (d) All of these

Direction *Answer the questions from 70-74 on the following case.*

Combination of Errors

Maximum absolute error in the sum or difference of two quantities is equal to sum of the absolute error in the individual quantities, i.e. $Z = A + B$, then $\pm \Delta Z = \pm \Delta A \pm \Delta B$

Maximum fractional error in a product or division of quantities is equal to the sum of fractional errors in the individual quantities

i.e. AB or $\dfrac{A}{B}$, then $\dfrac{\Delta Z}{Z} = \pm \dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}$

Two resistors of resistances $R_1 = 100 \pm 3\ \Omega$ and $R_2 = 200 \pm 4\Omega$ are connected (a) in series and (b) in parallel.

70. The percentage error in the value of R_1 is

(a) 3% (b) 4% (c) 6% (d) 0.3%

71. The fractional error in the value of R_2 is

(a) $\dfrac{1}{40}$ (b) $\dfrac{1}{50}$
(c) $\dfrac{1}{100}$ (d) $\dfrac{1}{200}$

72. Find the equivalent resistance of the series combination.

(a) $(250 \pm 7)\,\Omega$
(b) $(320 \pm 6)\,\Omega$
(c) $(300 \pm 7)\,\Omega$
(d) $(300 \pm 1)\,\Omega$

73. The percentage error in equivalent resistance in series combination is

(a) 2% (b) 2.3%
(c) 2.5% (d) 3%

74. Find the equivalent resistance of the parallel combination having error of 1.8 Ω.

(a) $(66 \pm 1)\,\Omega$ (b) $(66.7 \pm 1.8)\,\Omega$
(c) $(66.3 \pm 2)\,\Omega$ (d) $(67 \pm 3)\,\Omega$

Direction *Answer the questions from 75-79 on the following case.*

Dimensional analysis and its applications

The expression which shows how and which of the base quantities represent the dimensions of a physical quantity is called the dimensional formula of the given physical quantity. The recognition of concepts of dimensions, which guide the description of physical behaviour is of basic importance as only those physical quantities can be added or subtracted which have the same dimensions. A thorough understanding of dimensional analysis helps us in deducing certain relations among different physical quantities and checking the derivation, accuracy and dimensional consistency or homogeneity of various mathematical expressions. When magnitudes of two or more physical quantities are multiplied, their units should be treated in the same manner as ordinary algebraic symbols. We can cancel identical units in the numerator and denominator. The same is true for dimensions of a physical quantity. Similarly, physical quantities represented by symbols on both sides of a mathematical equation must have the same dimensions.

75. Statement I The method of dimensions analysis cannot validate the exact relationship between physical quantities in any equation.

Statement II It does not distinguish between the physical quantities having same dimensions.

Which of the following statement(s) is/are correct?

(a) Only I (b) I and II
(c) Only II (d) None of these

76. The quantity having same dimension as that of Planck's constant is

(a) work (b) linear momentum
(c) angular momentum (d) impulse

77. If speed v, acceleration A and force F, are considered as fundamental units, the dimension of Young's modulus will be

(a) $[v^{-4}A^{-2}F]$ (b) $[v^{-2}A^{2}F^{2}]$
(c) $[v^{-2}A^{2}F^{-2}]$ (d) $[v^{-4}A^{2}F^{1}]$

78. Given that, the amplitude of the scattered light is

(i) directly proportional to amplitude of incident light
(ii) directly proportional to the volume of the scattering dust particle
(iii) inversely proportional to its distance from the scattering particle and
(iv) dependent upon the wavelength λ of the light.

Then, the relation of intensity of scattered light with the wavelength is

(a) $\frac{1}{\lambda^2}$ (b) $\frac{1}{\lambda^4}$ (c) $\frac{1}{\lambda^6}$ (d) $\frac{1}{\lambda^7}$

79. Find the value of power of 60 J/min on a system that has 100 g, 100 cm and 1 min as the base units.

(a) 2.16×10^4 units (b) 2.16×10^6 units
(c) 3×10^4 units (d) 4×10^7 units

ANSWERS

Multiple Choice Questions

1. (b)	2. (c)	3. (c)	4. (a)	5. (d)	6. (c)	7. (b)	8. (a)	9. (a)	10. (b)
11. (b)	12. (a)	13. (a)	14. (b)	15. (a)	16. (b)	17. (c)	18. (a)	19. (d)	20. (a)
21. (a)	22. (c)	23. (a)	24. (b)	25. (d)	26. (c)	27. (a)	28. (c)	29. (a)	30. (a)
31. (a)	32. (d)	33. (a)	34. (d)	35. (a)	36. (b)	37. (a)	38. (b)	39. (b)	40. (b)
41. (a)	42. (b)	43. (b)	44. (d)	45. (d)	46. (c)	47. (b)	48. (c)	49. (d)	

Assertion-Reasoning MCQs

50. (d)	51. (c)	52. (c)	53. (a)	54. (b)	55. (b)	56. (d)	57. (d)	58. (a)	59. (a)

Case Based MCQs

60. (d)	61. (c)	62. (a)	63. (a)	64. (c)	65. (a)	66. (b)	67. (c)	68. (a)	69. (a)
70. (a)	71. (b)	72. (c)	73. (b)	74. (b)	75. (b)	76. (c)	77. (d)	78. (b)	79. (b)

SOLUTIONS

1. Time is the quantity which has same unit in all systems of unit, i.e. second. Other three quantities, i.e. mass, length and temperature have different units in different system of units.

2. The coefficient of thermal conductivity is given by

$$K = \frac{L}{A\Delta T}\frac{dQ}{dt}$$

where, L = length of conductor, A = area of conductor, ΔT = change in temperature and $\frac{dQ}{dt}$ = rate of flow of heat.

$\therefore$ Unit of $K = \frac{\text{metre}}{(\text{metre})^2 \times (\text{kelvin})} \times \text{watt}$

$= \text{Wm}^{-1}\text{K}^{-1}$

3. Given, damping force $\propto$ velocity

$$F \propto v \Rightarrow F = kv \quad \Rightarrow \quad k = \frac{F}{v}$$

Unit of $k = \frac{\text{Unit of } F}{\text{Unit of } v} = \frac{\text{kg - ms}^{-2}}{\text{ms}^{-1}} = \text{kg s}^{-1}$

4. To convert a measured value from one system to another system, we use

$$N_1 u_1 = N_2 u_2$$

where, N is numeric value and u is unit.

We get,

$$\therefore \quad 128 \cdot \frac{\text{kg}}{\text{m}^3} = N_2 \frac{50 \text{ g}}{(25 \text{ cm})^3}$$

$$\left[\because \text{density} = \frac{\text{mass}}{\text{volume}}\right]$$

$$\Rightarrow \frac{128 \times 1000 \text{ g}}{100 \times 100 \times 100 \text{ cm}^3} = \frac{N_2 \times 50 \text{ g}}{25 \times 25 \times 25 \text{ cm}^3}$$

$$\Rightarrow \quad N_2 = \frac{128 \times 1000 \times 25 \times 25 \times 25}{50 \times 100 \times 100 \times 100} = 40$$

5. Given, work done,

$$W = 10^{10} \text{g-cm}^2\text{s}^{-2}$$

which is in CGS system of units.

In SI unit, $W = 10^{10} \frac{\text{g}}{\text{s}^2}\text{cm}^2$

$$= 10^{10} \frac{(10^{-3}\text{kg})(10^{-4}\text{ m}^2)}{1\text{s}^2}$$

$$= 10^3 \text{kg-m}^2\text{s}^{-2}$$

6. We know that, 1 light year $= 9.46 \times 10^{11}$ m

= distance that light travels in 1 year with speed 3×10^8 m/s.

1 parsec = 3.08×10^{16} m

= Distance at which average radius of earth's orbit subtends an angle of 1 parsecond.

Here, year represent time.

7. We have, $\theta = 1°54' = (60 + 54)'$

$$= 114' = (114 \times 60)''$$

Since, $1'' = 4.85 \times 10^{-6}$ rad

$$= (114 \times 60)'' \times (4.85 \times 10^{-6}) \text{ rad}$$

$$= 3.33 \times 10^{-2} \text{ rad}$$

Also, diameter of earth, $b = 1.276 \times 10^7$ m

Hence, the earth-moon distance is given as

$$D = b/\theta = \frac{1.276 \times 10^7}{3.33 \times 10^{-2}} = 3.83 \times 10^8 \text{ m}$$

8. We know that, radius of atom, $r_a = 10^{-10}$ m

Radius of nucleus, $r_n = 10^{-15}$ m

$\therefore$ Ratio, $\frac{r_a}{r_n} = \frac{10^{-10}}{10^{-15}} = 10^5$

Ratio of volume $= \frac{\frac{4}{3}\pi r_a^3}{\frac{4}{3}\pi r_n^3} = \left(\frac{r_a}{r_n}\right)^3$

$$= (10^5)^3 = 10^{15}$$

9. All given measurements are correct upto two decimal places. As here 5.00 mm has the smallest unit and the error in 5.00 mm is least (commonly taken as 0.01 mm if not specified), hence 5.00 mm is most precise.

10. If student measure 3.50 cm, it means that there in an uncertainly of order 0.01 cm.

For vernier scale with 1 MSD

= 1mm and 9 MSD = 10 VSD

$\therefore$ LC of VC = 1 MSD − 1 VSD

$$= \frac{1}{10}\left(1 - \frac{9}{10}\right) = \frac{1}{100} \text{ cm}$$

11. Volume of block, $V = lbh$

The percentage error in the volume is given by

$$\frac{\Delta V}{V} \times 100 = \left(\frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta h}{h}\right) \times 100$$

$$= \left(\frac{0.02}{12.13} + \frac{0.01}{8.16} + \frac{0.01}{3.46}\right) \times 100$$

$$= \left(\frac{200}{1213} + \frac{100}{816} + \frac{100}{346}\right)$$

= 0.1649 + 0.1225 + 0.2890

= 0.58% (rounded off to two significant figures)

12. Arithmetic mean time of a oscillating simple pendulum $= \frac{\Sigma x_i}{N} = \frac{90 + 91 + 92 + 95}{4} = 92$ s

Mean deviation of a simple pendulum

$$= \frac{\Sigma |\bar{x} - x_i|}{N} = \frac{2 + 1 + 3 + 0}{4} = 1.5$$

Given, minimum division in the measuring clock, i.e. simple pendulum = 1 s. Thus, the reported mean time of a oscillating simple pendulum = (92 ± 2) s.

13. The mean period of oscillation of the pendulum,

$$T_{\text{mean}} = \frac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5}$$

$$= \frac{13.12}{5} = 2.624 = 2.62 \text{ s}$$

The absolute errors in the measurements are

$$\Delta T_1 = 2.63 \text{ s} - 2.62 \text{ s} = 0.01 \text{ s}$$
$$\Delta T_2 = 2.56 \text{ s} - 2.62 \text{ s} = -0.06 \text{ s}$$
$$\Delta T_3 = 2.42 \text{ s} - 2.62 \text{ s} = -0.20 \text{ s}$$
$$\Delta T_4 = 2.71 \text{ s} - 2.62 \text{ s} = 0.09 \text{ s}$$
$$\Delta T_5 = 2.80 \text{ s} - 2.62 \text{ s} = 0.18 \text{ s}$$

The arithmetic mean of all the absolute errors is

$$\Delta T_{\text{mean}} = \sum_{i=1}^{5} \frac{|\Delta T_i|}{5}$$

$$= [(0.01 + 0.06 + 0.20 + 0.09 + 0.18)] / 5$$

$$= 0.54/5 = 0.108 \simeq 0.11 \text{ s}$$

14. As we know, time period of oscillation is $T = 2\pi\sqrt{\frac{L}{g}}$

So, $g = 4\pi^2 L/T^2$

Therefore, relative error in g is

$(\Delta g/g) = (\Delta L/L) + 2(\Delta T/T)$

Given, $\Delta L = 1$ mm = 0.1 cm, $L = 20$ cm, $\Delta T = 1$ s and $T = 90$ s

$$\Rightarrow \frac{\Delta g}{g} = \frac{0.1}{20} + 2\left(\frac{1}{90}\right) = 0.027$$

Thus, the percentage error in g is

$$= \frac{\Delta g}{g} \times 100\%$$

$$= 0.027 \times 100\% = 2.7\% \simeq 3\%$$

15. Mean of the five observations,

$$\mu = \frac{80.0 + 80.5 + 81.0 + 81.5 + 82}{5}$$

$= \frac{405.0}{5} = 81$

$\therefore$ Mean error

$$= \frac{\left[\begin{array}{l}|80-\mu| + |80.5-\mu| + |81.0-\mu| \\ \qquad + |81.5-\mu| + |82-\mu|\end{array}\right]}{5}$$

$$= \frac{\left[\begin{array}{l}|80-81| + |80.5-81| + |81.0-81| \\ \qquad + |81.5-81| + |82-81|\end{array}\right]}{5}$$

$$= \frac{1 + 0.5 + 0 + 0.5 + 1}{5} = \frac{3}{5} = 0.6$$

$\therefore$ Mean % error $= \frac{0.6}{81} \times 100\% = 0.74\%$

16. The error in the measurement of mass 1.02 g is ± 0.01 g,

whereas that of another measurement 9.89 g is also ± 0.01 g.

$\therefore$ The relative error in 1.02 g

$= [\pm 0.01/1.02)] \times 100\% = \pm 0.98\% \cong \pm 1\%$

Similarly, the relative error in 9.89 g

$= [\pm 0.01/9.89] \times 100\% = \pm 0.1\%$

The relative errors in measurement of two masses are ± 1% and ± 0.1%.

17. $\because$ Density, $\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{L^3}$ or $\rho = \frac{M}{L^3}$

$\Rightarrow$ Error in density $\frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + \frac{3\Delta L}{L}$

So, maximum % error in measurement of ρ is

$$\frac{\Delta\rho}{\rho} \times 100 = \frac{\Delta M}{M} \times 100 + \frac{3\Delta L}{L} \times 100$$

or % error in density $= 1.5 + 3 \times 1$

% error = 4.5%

18. Kinetic energy, $K = \frac{1}{2}mv^2$

$$\therefore \quad \frac{\Delta K}{K} \times 100 = \frac{\Delta m}{m} \times 100 + \frac{2\Delta v}{v} \times 100$$

$$= 2\% + 2 \times 3\% = 8\%$$

19. Time period, $T = 2\pi\sqrt{\frac{l}{g}}$

or $\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}$

For 1 s ,

$$\Delta T = \frac{1}{2}\left(\frac{\Delta l}{l}\right)T = \frac{1}{2} \times 0.02 \times T = 0.01\,T$$

For a day, the pendulum loses,

$$\Delta T = 24 \times 60 \times 60 \times 0.01 = 864 \text{ s}$$

20. Given, length, $l = (16.2 \pm 0.1)$ cm

Breadth, $b = (10.1 \pm 0.1)$ cm

Area, $A = l \times b$

$= (16.2 \text{ cm}) \times (10.1 \text{ cm}) = 163.62 \text{ cm}^2$

Rounding off to three significant digits,

area, $A = 164 \text{ cm}^2$

$$\frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b} = \frac{0.1}{16.2} + \frac{0.1}{10.1}$$

$$= \frac{1.01 + 1.62}{16.2 \times 10.1} = \frac{2.63}{163.62}$$

$$\Rightarrow \quad \Delta A = A \times \frac{2.63}{163.62} = 163.62 \times \frac{2.63}{163.62}$$

$$= 2.63 \text{ cm}^2 \approx 3 \text{ cm}^2$$

(By rounding off to one significant figure)

$\therefore$ Area, $A = A \pm \Delta A = (164 \pm 3) \text{ cm}^2$

21. Given, $P = \frac{a^3 b^2}{cd}$, $\frac{\Delta a}{a} \times 100\% = 1\%$,

$\frac{\Delta b}{b} \times 100\% = 2\%$, $\quad \frac{\Delta c}{c} \times 100\% = 3\%$

and $\frac{\Delta d}{d} \times 100\% = 4\%$

$\therefore$ % error in $P = \left(\frac{\Delta P}{P} \times 100\right)\%$

$$= \left(\frac{3\Delta a}{a} + \frac{2\Delta b}{b} + \frac{\Delta c}{c} + \frac{\Delta d}{d}\right) \times 100\%$$

$$= \left(\begin{array}{l}3\frac{\Delta a}{a} \times 100\% + 2\frac{\Delta b}{b} \times 100\% \\ \qquad + \frac{\Delta c}{c} \times 100\% + \frac{\Delta d}{d} \times 100\%\end{array}\right)$$

$= 3 \times 1\% + 2 \times 2\% + 3\% + 4\% = 14\%$

22. Given, $z = \frac{a^2 b^{2/3}}{\sqrt{c}\, d^3}$

According to question,

% error in $z = (2)\%$ error in $a + \left(\frac{2}{3}\right)\%$ error in $b + \left(\frac{1}{2}\right)\%$ error in $c + (3)\%$ error in d

$$\frac{\Delta z}{z} = 2\frac{\Delta a}{a} + \frac{2}{3}\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + 3\frac{\Delta d}{d}$$

$$= 2 \times 2\% + \frac{2}{3} \times 1.5\% + \frac{1}{2} \times 4\% + 3 \times 2.5\%$$

$$= 14.5\%$$

23. The reliable digit plus the first uncertain digit is known as significant figures.
For the number 23.023, all the non-zero digits are significant, hence 5.
For the number 0.0003, number is less than 1, the zero(s) on the right of decimal point but to the left of the first non-zero digit are not significant, hence 1.
For the number 2.1×10^{-3}, significant figures are 2.

24. By adding 3.8×10^{-6} and 42×10^{-6}, we get
$= 45.8 \times 10^{-6} = 4.58 \times 10^{-5}$
As least number of decimal figures in given values is 1, so we round off the result to 4.6×10^{-5}.

25. The number 5.355 rounded off to three significant figures becomes 5.36, since preceding digit of 5 is odd, hence it is raised by 1.
On other hand, the number 5.345 rounded off to three significant figures becomes 5.34. Since, the preceding digit of 5 is even.

26. In this question, density should be reported to two significant figures.

$$\text{Density} = \frac{4.237\text{g}}{2.5\ \text{cm}^3} = 1.6948$$

As rounding off the number, we get density $= 1.7\ \text{g cm}^{-3}$

27. Dimensions of torque,

$$\tau = \mathbf{F} \times \mathbf{r} = [\text{MLT}^{-2}]\,[\text{L}] = [\text{ML}^2\text{T}^{-2}]$$

28. Dimensions of given quantities are
l = inductance = $[\text{M}^1\ \text{L}^2\ \text{T}^{-2}\ \text{A}^{-2}]$
r = resistance = $[\text{M}^1\ \text{L}^2\ \text{T}^{-3}\ \text{A}^{-2}]$
c = capacitance = $[\text{M}^{-1}\ \text{L}^{-2}\ \text{T}^4\ \text{A}^2]$
v = voltage = $[\text{M}^1\ \text{L}^2\ \text{T}^{-3}\ \text{A}^{-1}]$
So, dimensions of $\frac{l}{rcv}$ are

$$\left[\frac{l}{rcv}\right] = \frac{[\text{M L}^2\ \text{T}^{-2}\ \text{A}^{-2}]}{[\text{M}^1\ \text{L}^2\ \text{T}^{-2}\ \text{A}^{-1}]} = [\text{A}^{-1}]$$

29. According to ideal gas equation, i.e. $pV = nRT$, where n is the number of moles of gases.

$\therefore$ Universal gas constant, $R = \frac{(p)(V)}{(n)(T)}$

$$\text{Dimensional formula of } R = \frac{[\text{ML}^{-1}\ \text{T}^{-2}]\,[\text{L}^3]}{[\text{mol}]\,[\text{K}]} = [\text{ML}^2\text{T}^{-2}\ \text{mol}^{-1}\text{K}^{-1}]$$

30. I. Energy density $= \frac{\text{Energy}}{\text{Volume}} = \frac{[\text{ML}^2\text{T}^{-2}]}{[\text{L}^3]} = [\text{ML}^{-1}\text{T}^{-2}]$

II. Refractive index has no dimensions.
III. Dielectric constant has no dimensions.
IV. Young's modulus,

$$Y = \frac{Fl}{A\Delta l} = \frac{[\text{MLT}^{-2}][\text{L}]}{[\text{L}]^2[\text{L}]} = [\text{ML}^{-1}\text{T}^{-2}]$$

V. Magnetic field,

$$B = \frac{F}{Il} = \frac{[\text{MLT}^{-2}]}{[\text{A}][\text{L}]} = [\text{MT}^{-2}\text{A}^{-1}]$$

31. In this question, it is given that P, Q and R are having different dimensions, hence they cannot be added or subtracted, so we can say that (a) is not meaningful. We cannot say about the dimension of product of these quantities, hence (b), (c) and (d) may be meaningful.

32. Given, $U = \frac{A\sqrt{x}}{x+B}$
Dimensions of U = Dimensions of potential energy $= [\text{ML}^2\text{T}^{-2}]$
According to the principle of homogeneity,
Dimensions of B = Dimensions of $x = [\text{M}^0\text{LT}^0]$
$\therefore$ Dimensions of A

$$= \frac{\text{Dimensions of } U \times \text{Dimensions of } (x+B)}{\text{Dimensions of } \sqrt{x}}$$

$$= \frac{[\text{ML}^2\text{T}^{-2}]\,[\text{M}^0\text{LT}^0]}{[\text{M}^0\text{L}^{1/2}\text{T}^0]} = [\text{ML}^{5/2}\ \text{T}^{-2}]$$

Hence, dimensions of AB

$$= [\text{ML}^{5/2}\ \text{T}^{-2}]\,[\text{M}^0\text{LT}^0] = [\text{ML}^{7/2}\text{T}^{-2}]$$

33. According to question, $[X]$ = Dimensions of capacitance $= [\text{M}^{-1}\text{L}^{-2}\text{T}^2\text{Q}^2]$
and $[Z]$ = Dimensions of magnetic induction. $= [\text{MT}^{-1}\text{Q}^{-1}]$

Given, $X = 3YZ^2$,

$\therefore \quad [Y] = \dfrac{[X]}{[Z^2]}$

$\Rightarrow \quad [Y] = \dfrac{[M^{-1}L^{-2}T^2Q^2]}{[M^2T^{-2}Q^{-2}]} = [M^{-3}L^{-2}T^4Q^4]$

34. Given, $v = at + \dfrac{b}{t+c}$

Since, LHS is equal to velocity, so at and $\dfrac{b}{t+c}$ must have the dimensions of velocity.

$\therefore at = v$ or $a = \dfrac{v}{t} \Rightarrow [a] = \dfrac{[LT^{-1}]}{[T]} = [LT^{-2}]$

Now, c = time ($\because$ quantities are added)

$\therefore \quad c = t$

$[c] = [T]$

Now, $\dfrac{b}{t+c} = v$

$\therefore \quad b = v \times \text{time}$

$[b] = [LT^{-1}][T] = [L]$

35. The dimensions of LHS of each relation is [L],therefore the dimensions of RHS should be [L] as per the principle of homogeneity and the argument of the trigonometrical function, i. e. angle should be dimensionless.

(a) As $\dfrac{2\pi t}{T}$ is dimensionless, therefore dimensions of RHS = [L]. Thus, this formula is correct.

(b) Dimensions of RHS

$= [L]\sin[LT^{-1}][T] = [L]\sin[L]$

As angle is not dimensionless here, therefore, this formula is incorrect.

(c) Dimensions of RHS

$= \dfrac{[L]}{[T]}\sin\dfrac{[T]}{[L]} = [LT^{-1}]\sin[TL^{-1}]$

As angle is not dimensionless here, therefore this formula is incorrect.

$\therefore$ Thus, the correct formula on the dimensional ground is option (a).

36. Let Young's modulus is related to speed, area and force, as $Y = F^xA^yV^z$

Substituting dimensions, we have

$[ML^{-1}T^{-2}] = [MLT^{-2}]^x[L^2]^y[LT^{-1}]^z$

Comparing power of similar quantities, we have

$x = 1,\ x + 2y + z = -1$ and $-2x - z = -2$

Solving these, we get $x = 1,\ y = -1,\ z = 0$

So, $[Y] = [FA^{-1}V^0]$

37. Given, critical velocity of liquid flowing through tube is expressed as , $v_c \propto \eta^x \rho^y r^z$

Coefficient of viscosity of liquid, $\eta = [ML^{-1}T^{-1}]$

Density of liquid, $\rho = [ML^{-3}]$

Radius of a tube, $r = [L]$

Critical velocity of liquid, $v_c = [M^0LT^{-1}]$

$\Rightarrow \quad [M^0LT^{-1}] = [ML^{-1}T^{-1}]^x[ML^{-3}]^y[L]^z$

$[M^0LT^{-1}] = [M^{x+y}L^{-x-3y+z}T^{-x}]$

Comparing powers of M, L and T, we get

$x + y = 0,\ -x - 3y + z = 1,\ -x = -1$

On solving above equations, we get

$x = 1,\ y = -1,\ z = -1$

38. $\because$ Density $= \dfrac{\text{Mass}}{\text{Volume}}$

$\therefore$ Dimensions of density $= \dfrac{[M]}{[L]^3} = [ML^{-3}]$

Given, $n_1 = 10,\ M_1 = 1\text{ g},\ L_1 = 1\text{ cm}$,

In new system, $n_2 = ?,\ M_2 = 100\text{ g},\ L_2 = 10\text{ cm}$

So, conversion of 10g cm^{-3} (n_1) into new system

$$n_2 = n_1 \times \left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]^{-3}$$

$$= 10 \times \left(\frac{1}{100}\right)\left(\frac{1}{10}\right)^{-3}$$

$$= 10 \times \frac{1}{100} \times 10 \times 10 \times 10$$

$$= 100 \text{ units}$$

39. As, dimensional formula of force $= [MLT^{-2}]$

$n_1 = 36,\ M_1 = 1\text{ kg},\ L_1 = 1\text{ m},\ T_1 = 1\text{ min} = 60\text{ s}$

$n_2 = ?,\ M_2 = 1\text{ g},\ L_2 = 1\text{ cm},\ T_2 = 1\text{ s}$

So, conversion of 36 units into CGS system,

i. e. $n_2 = n_1\left[\dfrac{M_1}{M_2}\right]^a\left[\dfrac{L_1}{L_2}\right]^b\left[\dfrac{T_1}{T_2}\right]^c$

$\Rightarrow \quad n_2 = n_1\left[\dfrac{1\text{ kg}}{1\text{g}}\right]^1\left[\dfrac{1\text{ m}}{1\text{ cm}}\right]^1\left[\dfrac{1\text{ min}}{1\text{ s}}\right]^{-2}$

$= 36\left[\dfrac{1000\text{ g}}{1\text{ g}}\right]\left[\dfrac{100\text{ cm}}{1\text{ cm}}\right]^1\left[\dfrac{60\text{ s}}{1\text{ s}}\right]^{-2} = 10^3$ dyne

40. Solid angle, $d\Omega = \frac{dA}{r^2} = \frac{1\,\text{cm}^2}{(5\,\text{cm})^2}$

$= 0.04$ steradian

$= 4 \times 10^{-2}$ steradian

41. Given, $A = 2.5\ \text{ms}^{-1} \pm 0.5\ \text{ms}^{-1}$,

$B = 0.10\ \text{s} \pm 0.01\ \text{s}$

$x = AB = (2.5)(0.10) = 0.25\ \text{m}$

$$\frac{\Delta x}{x} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

$$= \frac{0.5}{2.5} + \frac{0.01}{0.10} = \frac{0.05 + 0.025}{0.25} = \frac{0.075}{0.25}$$

$\Delta x = 0.075 = 0.08$ m, rounding off to two significant figures.

$AB = (0.25 \pm 0.08)$ m

42. 100 years in seconds $= 100 \times 365 \times 24 \times 60 \times 60$ s. Error that may occur in the clock after these many seconds is 0.02 s

$\therefore$ Error in 1 s $= \frac{0.02\ \text{s}}{100 \times 365 \times 24 \times 60 \times 60}$

$= 10^{-11}$(approx.)

43. We know that, energy of radiation, $E = h\nu$

$$[h] = \frac{[E]}{[\nu]} = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$$

Dimensions of linear impulse = Dimension of momentum = $[MLT^{-1}]$

As we know that, linear impulse $J = \Delta P$

$\Rightarrow$ Angular impulse $= \tau dt = \Delta L$

= change in angular momentum

Hence, dimensions of angular impulse

= dimension of angular momentum

$= [ML^2T^{-1}]$

This is similar to the dimensions of Planck's constant h.

Dimensions of energy is $[ML^2T^{-2}]$.

44. Statement given in option (d) is incorrect and it can be corrected as

While dealing with atoms, kilogram is an inconvenient unit. In this case, there is an important standard unit of mass called unified atomic mass unit (u), which has been established for expressing the mass of atom.

Rest statements are correct.

45. When the relative error is expressed in percentage, we call it as percentage error.

Thus, statement (d) is incorrect while all other statements regarding measurement of a quantity are correct.

46. The correct relation for time period of simple pendulum is $T = 2\pi(l/g)^{1/2}$.

So, the given relation is numerically incorrect as the factor of 2π is missing.

But, it is dimensionally correct.

47. Capacitance,

$$C = \frac{Q}{V} = \frac{Q^2}{W} = (\text{coulomb})^2\,\text{joule}^{-1}$$

Magnetic induction,

$$B = \frac{F}{il} = \frac{\text{newton}}{\text{ampere} \times \text{metre}}$$

$= (\text{newton})\ (\text{ampere})^{-1}(\text{metre})^{-1}$

Inductance, $L = \frac{e}{dI/dt} = \frac{\text{volt}}{\text{ampere /second}}$

$= \text{volt-second}\ (\text{ampere})^{-1}$

Resistance, $R = \frac{V}{I} = \frac{\text{volt}}{\text{ampere}}$

$= \text{volt}\ (\text{ampere})^{-1}$

Hence, A $\rightarrow$ s, B $\rightarrow$ r, C $\rightarrow$ q and D $\rightarrow$ p.

48. Are is also unit of area, 1 are $= 10^2\ \text{m}^2$

Atmospheric pressure is measured in SI unit of bar.

1 bar $= 1.013 \times 10^5\ \text{N/m}^2 = 1.013 \times 10^5$ Pa

Carat is the unit of mass.

i.e. 1 carat = 200 mg

Hence, A $\rightarrow$ r, B $\rightarrow$ q and C $\rightarrow$ p.

49. Dimensions of Pa-s is $[ML^{-1}T^{-2}] \cdot [T]$

$= [ML^{-1}T^{-1}]$

Dimensions of Nm K^{-1} is

$[MLT^{-2}][L][K^{-1}] = [ML^2T^{-2}K^{-1}]$

Dimensions of J-kg^{-1} K^{-1} is

$[ML^2T^{-2}][M^{-1}][K^{-1}] = [L^2T^{-2}K^{-1}]$

Dimensions of Wbm^{-1} K^{-1} is

$[ML^2T^{-2}A^{-1}][L^{-1}][K^{-1}] = [MLT^{-2}A^{-1}K^{-1}]$

Here, W is for weber, unit of magnetic flux BA with dimensions

$$\frac{[MLT^{-2}]}{[AT]\,[LT^{-1}]}[L^2] = ML^2T^{-2}A^{-1}$$

Hence, $A \rightarrow r$, $B \rightarrow s$, $C \rightarrow p$ and $D \rightarrow q$.

50. The unit chosen for measuring any physical quantity, should be easily reproducible, i.e. replicas of the unit should be available easily.

Also, unit should not change with changing physical conditions like temperature, pressure, etc.

Therefore, A is false and R is also false.

51. Barn is used in nuclear physics for measuring the cross-sectional area of nuclei.

One barn is equal to 10^{-28} m^2.

Therefore, A is true but R false.

52. Changing the unit of the measurement, the numerical value of the quantity also changes. For example, let the length of scale be 1 m.

Its value in CGS unit is 100 cm.

Therefore, the numerical value changes.

Also, we can say that from the above example smaller the unit of measurement, greater is its numerical value.

Therefore, A is true but R is false.

53. Parallax method is used for measuring distances of nearby stars only.

If D is a distance of a far away star from Earth, then $D = \frac{b}{\theta}$

where, θ is called parallactic angle and b is the distance between the two different positions on Earth from where the star is being observed.

$\therefore$ With increase in the distance of star, parallactic angle becomes too small to be measured accurately.

Therefore, both A and R are true and R is the correct explanation of A.

54. Accuracy of the measurement is the measure of how close the measured value is to the true value.

So, the greater the significant figures in the digit, greater will be its accuracy.

Since, 0.70 m has more significant figure (i.e., 2) as compare to 0.7 m (i.e., 1). So, it will be more accurate.

Also, in general, the last digit is not accurately known in every measurement.

Therefore, both A and R are true but R is not the correct explanation of A.

55. Random errors are those errors, which occur irregularly and hence are random with respect to sign and size.

These can arise due to random and unpredictable fluctuations in experimental conditions, personal (unbiased) errors by the observer taking readings, etc.

Therefore, both A and R are true but R is not the correct explanation of A.

56. In all mathematical operations, the errors are of additive nature.

When a quantity appears with a power n greater than one in an expression, its error contribution to the final result increases n times.

So, quantities with higher power in the expression should be measured with maximum accuracy.

Therefore, A is false and R is also false.

57. The arguments of special functions, such as the trigonometric, logarithmic and exponential functions must be dimensionless.

A pure number, ratio of similar physical quantities, such as angle as the ratio (length/length), refractive index as the ratio of (speed of light in vacuum/speed of light in medium), etc. has no dimensions.

Therefore, A is false and R is also false.

58. Relative density of a fluid $= \frac{\text{Density of fluid}}{\text{Density of water}}$

As the density of any substance has same units, hence relative density is dimensionless.

Therefore, both A and R are true and R is the correct explanation of A.

59. The method of dimensions can only test the dimensional validity but not the exact relationship between physical quantities in any equation.

This is because it does not distinguish between the physical quantities having same dimensions.

Therefore, both A and R are true and R is the correct explanation of A.

60. A screw gauge and a spherometer can be used to measure length accurately as less as 10^{-5} m.

61. Spring oscillator cannot be used to measure time intervals.

62. Given, length, $l = 5$ cm

Now, checking the errors with each options one-by-one, we get

$$\Delta l_1 = 5 - 4.9 = 0.1 \text{ cm}$$
$$\Delta l_2 = 5 - 4.805 = 0.195 \text{ cm}$$
$$\Delta l_3 = 5.25 - 5 = 0.25 \text{ cm}$$
$$\Delta l_4 = 5.4 - 5 = 0.4 \text{ cm}$$

Error Δl_1 is least.

Hence, 4.9 cm is most precise or accurate.

63. Area of rectangle, $A = \text{Length} \times \text{Breadth}$

So, $A = lb = 10.5 \times 2.1 = 22.05 \text{ cm}^2$

Minimum possible measurement of scale = 0.1 cm.

So, area measured by scale = 22.0 cm^2.

64. Magnification in time $= \dfrac{\text{Age of mankind}}{\text{Age of universe}}$

$$= \frac{10^6}{10^{10}} = 10^{-4}$$

Apparent age of mankind $= 10^{-4} \times 1$ day

$$= 10^{-4} \times 86400 \text{ s}$$
$$= 8.64 \text{ s} \approx 8.6 \text{ s}$$

65. As, we know that, the terminal or trailing zero(s) in a number without a decimal point are not significant. So, 4700 m has two significant figures.

66. Every number is expressed as $a \times 10^b$, where a is a number between 1 & 10 and b is any positive or negative exponent (or power) of 10.

67. There are 3 significant figures in the measured mass whereas there are only 2 significant figures in the measured volume. Hence, the density should be expressed to only 2 significant figures.

Density $= \dfrac{5.74}{1.2} = 4.8 \text{ g cm}^{-3}$

68. There is no change in number of significant figures on changing the units. For it, the convention is that we write,

$$4700 \text{ m} = 4.700 \times 10^3 \text{ m}$$

This convention ensures no change in number of significant numbers.

69. Following rules of significant figures are

I. All the non-zero digits are significant.

II. All the zeroes between two non-zero digits are significant, no matter where the decimal point is, if at all.

III. The terminal or trailing zero(s) in a number without a decimal point are not significant. Thus, 123 m = 12300 cm = 123000 mm has three significant figures, the trailing zero(s) being not significant.

70. Given, $R_1 = (100 \pm 3)\,\Omega$

$$\therefore \frac{\Delta R_1}{R_1} \times 100 = \frac{3}{100} \times 100 = 3\%$$

71. Given, $R_2 = (200 \pm 4)\,\Omega$

$$\therefore \frac{\Delta R_2}{R_2} = \frac{4}{200} = \frac{1}{50}$$

72. The equivalent resistance of series combination,

i.e. $R_S = R_1 + R_2 = (100 \pm 3)\,\Omega + (200 \pm 4)\,\Omega$
$= (300 \pm 7)\,\Omega$

73. As, $R_S = (300 \pm 7)\,\Omega$

$$\therefore \frac{\Delta R_S}{R_S} \times 100 = \frac{7}{300} \times 100 = 2.3\%$$

74. The equivalent resistance of parallel combination,

$$R' = \frac{R_1 R_2}{R_1 + R_2}$$
$$= \frac{200}{3} = 66.7\,\Omega$$

Given, $\Delta R' = 1.8\,\Omega$

$\therefore \quad R' = (66.7 \pm 1.8)\,\Omega$

75. The method of dimensions can only test the dimensional validity but not the exact relationship between physical quantities in any equation.

This is because, it does not distinguish between the physical quantities having same dimensions.

76. Planck's constant, $h = \frac{E}{\nu}$

So, dimensions of $h = \left[\frac{ML^2T^{-2}}{T^{-1}}\right] = [ML^2T^{-1}]$

Angular momentum, $L = mvr$

Dimensions of $L = [M]\,[LT^{-1}]\,[L] = [ML^2T^{-1}]$

Work, W = Force × Displacement

$\therefore$ Dimensions of $W = [MLT^{-2}] \times [L] = [ML^2T^{-2}]$

Linear momentum, p = Mass × Velocity

Dimensions of $p = [M][LT^{-1}] = [MLT^{-1}]$

$$\text{Impulse, } I = \frac{\text{Force}}{\text{Time}}$$

Dimensions of $I = \frac{[MLT^{-2}]}{[T]} = [MLT^{-3}]$

Hence, only angular momentum has same dimensions as that of Planck's constant.

77. Dimensions of speed $[v] = [LT^{-1}]$

Dimensions of acceleration $[A] = [LT^{-2}]$

Dimensions of force $[F] = [MLT^{-2}]$

Dimensions of Young modulus $[Y] = [ML^{-1}T^{-2}]$

Let dimensions of Young's modulus is expressed in terms of speed, acceleration and force as

$$[Y] = [v]^{\alpha}[A]^{\beta}[F]^{\gamma} \quad \text{...(i)}$$

Then substituting dimensions in terms of M, L and T, we get

$$[ML^{-1}T^{-2}] = [LT^{-1}]^{\alpha}[LT^{-2}]^{\beta}[MLT^{-2}]^{\gamma}$$
$$= [M^{\gamma}L^{\alpha+\beta+\gamma}\,T^{-\alpha-2\beta-2\gamma}]$$

Now comparing powers of basic quantities on both sides, we get

$$\gamma = 1$$
$$\alpha + \beta + \gamma = -1$$

and $-\alpha - 2\beta - 2\gamma = -2$

Solving these, we get

$$\alpha = -4,\ \beta = 2,\ \gamma = 1$$

Substituting the values of α, β and γ in Eq. (i), we get

$$[Y] = [v^{-4}A^2F^1]$$

78. According to the question, the expression for the scattered amplitude of light (A_s) in terms of amplitude of incident light (A_i), volume (V), distance from scattering particle (x) and wavelength (λ) can be given as

$\therefore \quad A_s = kA_i^1\,V^1x^{-1}\lambda^d$

where, k is the constant of proportionality.

Writing the dimensions on both sides of the above equation, we get

$$[L] = [L]\,[L^3]\,[L^{-1}]\,[L^d] = [L^{3+d}]$$

Comparing the powers of L on both sides, we get

or $\quad 1 = 3 + d$

or $\quad d = -2$

i.e. $\quad A_s \propto \frac{1}{\lambda^2}$

But, intensity $(I_s) \propto$ [amplitude $(A_s)]^2$

$\therefore \quad I_s \propto \frac{1}{\lambda^4}$

79. Given, power, $P_1 = \frac{\text{Work done}}{\text{Time taken}}$

$$= \frac{60\text{ J}}{1\text{min}} = \frac{60\text{ J}}{60\text{ s}} = 1\text{ W or kg-m}^2\text{s}^{-3}$$

which is the SI unit of power.

Given, $P_1 = 1$ W, $M_1 = 1\text{kg} = 1000$ g

$L_1 = 1$ m $= 100$ cm, $T_1 = 1$ s

In new system, $P_2 = ?$, $M_2 = 100$ g, $L_2 = 100$ cm, $T_2 = 1$ min $= 60$ s

$\therefore$ Conversion of 60 J per min or 1W in a new system, i.e.

$$P_2 = P_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

Now, [power] $= [ML^2T^{-3}]$

So, $\quad a = 1,\ b = 2$

and $\quad c = -3$

$$\Rightarrow \quad P_2 = 1\left[\frac{1000}{100}\right]^1\left[\frac{100}{100}\right]^2\left[\frac{1}{60}\right]^{-3}$$

$$= 2.16 \times 10^6 \text{ units}$$

$\therefore$ 60 J min^{-1} $= 2.16 \times 10^6$ new units of power